Powers and Exponents

Exponent
$$3^4 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{\text{Base common factors}} = 81$$

The **exponent** tells you how many times to use the **base** as a factor.

EXAMPLE 1 Write 6^3 as a product of the same factor.

The base is 6. The exponent 3 means that 6 is used as a factor 3 times. $6^3 = 6 \cdot 6 \cdot 6$

EXAMPLE 2 Evaluate 54.

$$5^4 = 5 \cdot 5 \cdot 5 \cdot 5$$
$$= 625$$

EXAMPLE 3. Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ in exponential form.

The base is 4. It is used as a factor 5 times, so the exponent is 5. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4^5$

EXERCISES

Write each power as a product of the same factor.

3.
$$9^2$$

Evaluate each expression.

Write each product in exponential form.



Order of Operations

Use the **order of operations** to evaluate numerical expressions.

- 1. Do all operations within grouping symbols first.
- 2. Evaluate all powers before other operations.
- 3. Multiply and divide in order from left to right.
- 4. Add and subtract in order from left to right.

EXAMPLE 1 Evaluate $(10-2)-4\cdot 2$.

$$(10-2)-4\cdot 2=8-4\cdot 2$$
 Subtract first since $10-2$ is in parentheses.
= $8-8$ Multiply 4 and 2.
= 0 Subtract 8 from 8.

EXAMPLE 2 Evaluate $8 + (1+5)^2 \div 4$.

$$8 + (1 + 5)^{2} \div 4 = 8 + 6^{2} \div 4$$

$$= 8 + 36 \div 4$$

$$= 8 + 9$$

$$= 17$$

First, add 1 and 5 inside the parentheses. Find the value of 6².

Divide 36 by 4. Add 8 and 9.

EXERCISES

Evaluate each expression.

1.
$$(1 + 7) \times 3$$

3.
$$5 + 4 \cdot 3$$

4.
$$(40 \div 5) - 7 + 2$$

5.
$$35 \div 7(2)$$

6.
$$3 \times 10^3$$

7.
$$45 \div 5 + 36 \div 4$$

8.
$$42 \div 6 \times 2 - 9$$

9.
$$2 \times 8 - 3^2 + 2$$

10.
$$5 \times 2^2 + 32 \div 8$$

10.
$$5 \times 2^2 + 32 \div 8$$
 11. $3 \times 6 - (9 - 8)^3$

12.
$$3.5 \times 10^2$$

Algebra: Variables and Expressions

To evaluate an algebraic expression you replace each variable with its numerical value, then use the order of operations to simplify.

EXAMPLE

Evaluate 6x - 7 if x = 8.

$$6x - 7 = 6(8) - 7$$
$$= 48 - 7$$

Replace x with 8. Use the order of operations.

Subtract 7 from 48.

EXAMPLE 2 Evaluate 5m - 3n if m = 6 and n = 5.

$$5m - 3n = 5(6) - 3(5)$$
$$= 30 - 15$$

Replace m with 6 and n with 5.

Use the order of operations.

= 15

Subtract 15 from 30.

Evaluate $\frac{ab}{3}$ if a = 7 and b = 6.

$$\frac{ab}{3} = \frac{(7)(6)}{3}$$

Replace a with 7 and b with 6.

$$=\frac{42}{3}$$

The fraction bar is like a grouping symbol.

Divide.

EXAMPLE 4 Evaluate $x^3 + 4$ if x = 3.

$$x^3 + 4 = 3^3 + 4$$

Replace x with 3.

$$= 27 + 4$$

Use the order of operations.

Add 27 and 4.

EXERCISES

Evaluate each expression if a = 4, b = 2, and c = 7.

2.
$$5b^3$$

4.
$$5 + 6c$$

5.
$$\frac{ab}{8}$$

6.
$$2a - 3b$$

7.
$$\frac{b^4}{4}$$

8.
$$c-a$$

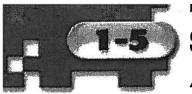
9.
$$20 - bc$$

11.
$$ac - 3b$$

12.
$$6a^2$$

14.
$$6a - b$$

15.
$$ab - c$$



Algebra: Equations

- An equation is a sentence in mathematics that contains an equals sign.
- The solution of an equation is the value that when substituted for the variable makes the equation

EXAMPLE 1 Solve 23 + y = 29 mentally.

 $23 + \nu = 29$ Write the equation.

23 + 6 = 29You know that 23 + 6 is 29.

29 = 29Simplify.

The solution is 6.

EXAMPLE

What value of x is a solution of x + 8 = 26?

A. 16

C. 18

D. 19

Substitute each value for x to determine which makes the left side of the equation equivalent to the right side.

Replace x with 16.

Replace x with 17.

Replace x with 18.

x + 8 = 26

$$x + 8 = 26$$

$$x + 8 = 26$$

 $18 + 8 = 26$

16 + 8 = 26 $24 \neq 26$ false

$$17 + 8 = 26$$

 $25 \neq 26$ false

$$26 = 26$$
 true

The value of 18 makes the equation true. So, the answer is C.

EXERCISES

Solve each equation mentally.

1.
$$k + 7 = 15$$

2.
$$g - 8 = 20$$

3.
$$6y = 24$$

4.
$$\frac{a}{3} = 9$$

5.
$$\frac{x}{6} = 9$$

6.
$$8 + r = 24$$

7.
$$12 \cdot 8 = h$$

8.
$$n \div 11 = 8$$

9.
$$48 \div 12 = x$$

10.
$$h - 12 = 24$$

11.
$$19 + y = 28$$

12.
$$9f = 90$$

Name the number that is the solution of the given equation.

13.
$$27 - h = 10$$
; 7, 17, 27

14.
$$n \div 11 = 4$$
; 44, 55, 66



Adding Integers

For integers with the same sign:

- the sum of two positive integers is positive.
- the sum of two negative integers is negative.

For integers with different signs, subtract their absolute values. The sum is:

- positive if the positive integer has the greater absolute value.
- negative if the negative integer has the greater absolute value.

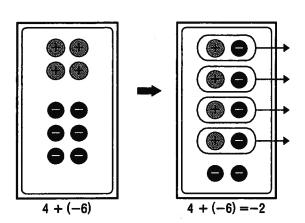
To add integers, it is helpful to use counters or a number line.

EXAMPLE

Find 4 + (-6).

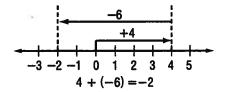
Method 1 Use counters.

Combine a set of 4 positive counters and a set of 6 negative counters on a mat.



Method 2 Use a number line.

- Start at 0.
- Move 4 units right.
- Then move 6 units left.



EXERCISES

Add.

1.
$$-5 + (-2)$$

$$2.8 + 1$$

$$3. -7 + 10$$

4.
$$16 + (-11)$$

5.
$$-22 + (-7)$$

6.
$$-50 + 50$$

7.
$$-10 + (-10)$$

8.
$$100 + (-25)$$

9.
$$-35 + -20$$

Evaluate each expression if a = 8, b = -8, and c = 4.

10.
$$a + 15$$

11.
$$b + (-9)$$

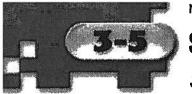
12.
$$a + b$$

13.
$$b + c$$

14.
$$-10 + c$$

15.
$$12 + b$$

DATE



Study Guide and Intervention

Subtracting Integers

To subtract an integer, add its opposite.

EXAMPLE 1 Find 6-9.

$$6 - 9 = 6 + (-9)$$

= -3

To subtract 9, add -9. Simplify.

EXAMPLE 2 Find -10 - (-12).

$$-10 - (-12) = -10 + 12$$
 To subtract -12, add 12.
= 2 Simplify.

Simplify.

EXAMPLE 3 Evaluate a - b if a = -3 and b = 7.

$$a - b = -3 - 7$$

= -3 + (-7)
= -10

Replace a with -3 and b with 7. To subtract 7, add -7. Simplify.

EXERCISES

Subtract.

2.
$$20 - (-6)$$

$$3. -10 - 4$$

4.
$$0 - 12$$

5.
$$-7 - 8$$

7.
$$-20 - (-5)$$

8.
$$-8 - (-6)$$

9.
$$25 - (-14)$$

10.
$$-75 - 50$$

12.
$$19 - (-10)$$

Evaluate each expression if m = -2, n = 10, and p = 5.

13.
$$m - 6$$

14.
$$9 - n$$

15.
$$p - (-8)$$

16.
$$p - m$$

17.
$$m - n$$

18.
$$-25 - p$$

Multiplying Integers

The product of two integers with different signs is negative.

The product of two integers with the same sign is positive.

EXAMPLE

Multiply 5(-2).

5(-2) = -10

The integers have different signs. The product is negative.

EXAMPLE 2

Multiply -3(7).

-3(7) = -21

The integers have different signs. The product is negative.

EXAMPLE

Multiply -6(-9).

-6(-9) = 54

The integers have the same sign. The product is positive.

EXAMPLE

Multiply $(-7)^2$.

 $(-7)^2 = (-7)(-7)$

There are 2 factors of -7. The product is positive.

EXAMPLE

Simplify -2(6c).

 $-2(6c)=(-2\cdot 6)c$

Associative Property of Multiplication.

=-12c

Simplify.

EXAMPLE 6

Simplify 2(5x).

 $2(5x) = (2 \cdot 5)x$

Associative Propery of Multiplication.

= 10x

Simplify.

EXERCISES

Multiply.

1. -5(8)

2. -3(-7)

3. 10(-8)

4. -8(3)

5. -12(-12)

6. $(-8)^2$

ALGEBRA Simplify each expression.

7. -5(7a)

8. 3(-2x)

9. 4(6f)

10. 7(6b)

11. -6(-3y)

12. 7(-8g)

ALGEBRA Evaluate each expression if a = -3, b = -4, and c = 5.

13. -2a

14. 9*b*

15. ab

16. -3ac

17. $-2c^2$

18. abc



Dividing Integers

The quotient of two integers with different signs is negative.

The quotient of two integers with the same sign is positive.

EXAMPLE 1 Divide $30 \div (-5)$.

$$30 \div (-5)$$

The integers have different signs.

$$30 \div (-5) = -6$$

The quotient is negative.

EXAMPLE 2 Divide $-100 \div (-5)$.

$$-100 \div (-5)$$

The integers have the same sign.

$$-100 \div (-5) = 20$$

The quotient is positive.

EXERCISES

Divide.

1.
$$-12 \div 4$$

2.
$$-14 \div (-7)$$

3.
$$\frac{18}{-2}$$

4.
$$-6 \div (-3)$$

5.
$$-10 \div 10$$

6.
$$\frac{-80}{-20}$$

7.
$$350 \div (-25)$$

8.
$$-420 \div (-3)$$

9.
$$\frac{540}{45}$$

10.
$$\frac{-256}{16}$$

ALGEBRA Evaluate each expression if d = -24, e = -4, and f = 8.

11.
$$12 \div e$$

12.
$$40 \div f$$

13.
$$d \div 6$$

14.
$$d \div e$$

15.
$$f \div e$$

16.
$$e^2 \div f$$

17.
$$\frac{-d}{e}$$

18.
$$ef \div 2$$

19.
$$\frac{f^2}{e^2}$$

20.
$$\frac{de}{f}$$

Solving Addition and Subtraction Equations

Remember, equations must always remain balanced. If you subtract the same number from each side of an equation, the two sides remain equal. Also, if you add the same number to each side of an equation, the two sides remain equal.

EXAMPLE Solve x + 5 = 11. Check your solution.

x + 5 = 11Write the equation.

-5 = -5Subtract 5 from each side.

Simplify.

Check x + 5 = 11Write the equation.

 $6 + 5 \stackrel{?}{=} 11$ Replace x with 6.

 $11 = 11 \checkmark$ This sentence is true.

The solution is 6.

EXAMPLE 2 Solve 15 = t - 12. Check your solution.

15 = t - 12 Write the equation.

+12 = +12Add 12 to each side.

Simplify.

Check 15 = t - 12

Write the equation.

 $15 \stackrel{?}{=} 27 - 12$ Replace t with 27.

 $15 = 15 \checkmark$

This sentence is true.

The solution is 27.

EXERCISES

Solve each equation. Check your solution.

1.
$$h + 3 = 14$$

2.
$$m + 8 = 22$$

3.
$$p + 5 = 15$$

4.
$$17 = y + 8$$

$$5 m + 4 = -1$$

5.
$$w + 4 = -1$$
 6. $k + 5 = -3$

7.
$$25 = 14 + r$$

7.
$$25 = 14 + r$$
 8. $57 + z = 97$

9.
$$h - 3 = 6$$

10.
$$7 = c - 8$$

11.
$$j - 12 = 18$$

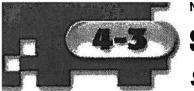
9.
$$b-3=6$$
 10. $7=c-5$ **11.** $j-12=18$ **12.** $v-4=18$

13.
$$-9 = w - 12$$
 14. $y - 8 = -12$ **15.** $14 = f - 2$ **16.** $23 = n - 12$

14.
$$y - 8 = -12$$

15.
$$14 = f - 2$$

16.
$$23 = n - 12$$



Solving Multiplication Equations

If each side of an equation is divided by the same non-zero number, the resulting equation is equivalent to the given one. You can use this property to solve equations involving multiplication and division.

EXAMPLE 1 Solve 45 = 5x. Check your solution.

45 = 5x

Write the equation.

 $\frac{45}{5} = \frac{5x}{5}$

Divide each side of the equation by 5.

9 = x

$$45 \div 5 = 9$$

Check 45 = 5x

Write the original equation.

45 2 5(9)

Replace x with 9. Is this sentence true?

 $45 = 45 \checkmark$

The solution is 9.

EXAMPLE 2 Solve -21 = -3y. Check your solution.

-21 = -3y Write the equation.

 $\frac{-21}{-3} = \frac{-3y}{-3}$ Divide each side by -3.

7 = y

$$-21 \div (-3) = 7$$

 $\mathbf{Check} - 21 = -3y$

Write the original equation.

 $-21 \stackrel{?}{=} -3(7)$

Replace y with 7. Is this sentence true?

-21 = -21

The solution is 7.

EXERCISES

Solve each equation. Then check your solution.

1.
$$8q = 56$$

2.
$$4p = 32$$

3.
$$42 = 6m$$

4.
$$104 = 13h$$

5.
$$-6n = 30$$

6.
$$-18x = 36$$
 7. $48 = -8y$ 8. $72 = -3b$

7.
$$48 = -8y$$

8.
$$72 = -3b$$

9.
$$-9a = -45$$

10.
$$-12m = -120$$
 11. $-66 = -11t$ 12. $-144 = -9r$

11.
$$-66 = -11$$

12.
$$-144 = -9r$$

13.
$$3a = 4.5$$

14.
$$2h = 3.8$$

15.
$$4.9 = 0.7k$$

16.
$$9.75 = 2.5z$$